

Numerical Modelling of Stochastic Resonance and Anomalous Diffusion in a Nonlinear Bistable System

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This paper presents a computational simulation of a microscopic particle suspended in a viscous fluid, described by the nonlinear Langevin equation. Four forces act on the particle simultaneously: friction (Stokes drag), thermal noise (white noise), a periodic force, and a force induced by the potential. By placing the particle in a symmetric double-well potential $U(x) = -x^2 + x^4$, we investigate two distinct physical phenomena.

First, classical and anomalous diffusion are characterised through Mean Squared Displacement (MSD) analysis over 500 particle trajectories. The logarithmic (log-log) scale reveals a diffusion exponent $\alpha < 1$, confirming subdiffusive behaviour over a certain time interval. According to Einstein's theory, MSD grows linearly with time. It is shown that velocity distributions in the stationary state conform to the Maxwell-Boltzmann distribution, confirming that our simulation is thermodynamically correct.

Second, we demonstrate Stochastic Resonance at the optimal noise amplitude ($\sigma = 0.45$). At this amplitude, the chaotic force instead assists the particle in crossing the potential barrier, and it begins synchronised transitions at $x = \pm 0.707$. At other amplitudes the particle would become stuck in the potential barrier. This phenomenon is also examined in two dimensions, where along the X-axis the particle's motion is constrained by the potential and obeys Stochastic Resonance, while along the Y-axis it maintains free Brownian diffusion.

These results show that in nonlinear systems, noise can amplify the signal rather than degrading it.